

# Chapter 3. Discrete Random Variables & Probability Dist.

## §3.1: Random Variables

Def: A random variable is a rule assigning a number to each outcome in a sample space.

Random variables are written with capital letters  $X$  or  $Y$  etc.

Lower case letters are used for non-random variables & particular outcomes

Idea: A non-random variable  $x$  is an unknown #  
A random variable  $X$  is not a single #  
it is multiple numbers with different prob.

Example: Flip two coins.  $X = \#$  of heads

$$P(X=0) = 1/4$$
$$P(X=1) = 1/2$$
$$P(X=2) = 1/4$$

Example: Roll two dice.  $X =$  difference between rolls

$$P(X=0) = 6/36$$
$$P(X=1) = 10/36$$
$$P(X=2) = 8/36$$
$$P(X=3) = 6/36$$
$$P(X=4) = 4/36$$
$$P(X=5) = 2/36$$

Example: Flip coin until H.  $X = \#$  flips required

$$P(X=1) = 1/2$$
$$P(X=2) = 1/4$$
$$P(X=3) = 1/8$$
$$\vdots$$
$$P(X=n) = 1/2^n$$

(Recall from MAT119:  $\sum_{n=1}^{\infty} 1/2^n = 1/2 + 1/4 + 1/8 + \dots = 1$ )

These were all examples of discrete random variables.

- Discrete random variables take values in either a finite (or countable) set.
- Continuous random variables take values in an interval.

Discrete random variables use sums "+" and " $\Sigma$ "  
Continuous random variables use integrals  $\int dx$

The simplest type of discrete random variable is the Bernoulli Random Variable.

Jacob Bernoulli  
1654-1705  
Swiss mathematician  
→ Discovered  $e$ !

A Bernoulli r.v. is  $X$  whose only possible values are 0 & 1

Usually we say  $\left\{ \begin{array}{l} 1 = \text{"success"} \\ 0 = \text{"failure"} \end{array} \right\}$  but it doesn't have to be that way...

Most examples we have done so far were secretly Bernoulli.

Given an event  $E$  we can define  $X$  as

$$X(x) = \begin{cases} 1 & \text{if } x \in E \\ 0 & \text{if } x \notin E \end{cases}$$

↳ i.e.  $X=1$  if  $E$  occurred

$X=0$  if  $E$  did not occur

Example: Roll a die. Let  $E = \{ \text{roll is } \leq 4 \}$

$$X = \begin{cases} 1 & \text{if } \leq 4 \\ 0 & \text{if } \geq 5 \end{cases}$$

$$P(X=1) = 4/6$$

$$P(X=0) = 2/6$$

Example: Roll two dice. Let  $E = \{ \text{sum is } \geq 10 \}$ .

$$X = \begin{cases} 1 & \text{if sum } \geq 10 \\ 0 & \text{if sum } \leq 9 \end{cases}$$

$$P(X=1) = 6/36$$

$$P(X=0) = 30/36$$